Math 1A: Calculus **Instructor: Alexander Paulin**

Handout: IVT, EVT, MVT

Discussions 201, 203 // 2018-10-22

Theorem (Intermediate Value Theorem)**.** *Let $a < b$ be real numbers and suppose f is a function that is* $\underset{\text{(adjective)}}{\underline{\hspace{3cm}}}$ *on the closed interval $[a, b]$. If d is any value strictly between* _______ *and* _______ *then there exists c in the interval (a, b) for which $f(c) = d$.*

Theorem (Extreme Value Theorem)**.** *Let $a < b$ be real numbers and suppose f is a function that is* $\underset{\text{(adjective)}}{\underline{\hspace{3cm}}}$ *on the closed interval $[a, b]$. Then f has an absolute maximum and an absolute minimum (possibly not unique) on the interval $[a, b]$.*

Theorem (Mean Value Theorem)**.** *Let $a < b$ be real numbers and suppose f is a function that is* $\underset{\text{(adjective)}}{\underline{\hspace{3cm}}}$ *on the closed interval $[a, b]$ and* $\underset{\text{(adjective)}}{\underline{\hspace{3cm}}}$ *on the open interval (a, b). Then there exists c in the interval (a, b) for which*

$$f'(c) = \frac{\hspace{3cm}}{\hspace{3cm}}$$

Corollary. *If $f'(x) = g'(x)$ on an interval (a, b), then $f - g$ is* $\underset{\text{(adjective)}}{\underline{\hspace{3cm}}}$ *on that interval. In particular, if $f'(x) = 0$ on an interval (a, b), then f is* $\underset{\text{(same adjective)}}{\underline{\hspace{3cm}}}$ *on (a, b).*

The following few exercises will show you why the assumptions in the above theorems are important.

Exercise 1. The function $f(x) = 1/x$ satisfies $f(-1) = -1$ and $f(1) = 1$. But even though $-1 < 0 < 1$, there is no c in the interval $(-1, 1)$ for which $f(c) = 0$. Why does this not contradict IVT?

Exercise 2. Does $f(x) = \arctan x$ have an absolute maximum, over its whole domain $\mathbb{R}$? Does $g(x) = x$ have an absolute minimum on the interval $(0, 1]$? Why does this not contradict EVT?

Exercise 3. Let $f(x) = |x|$. Then,

$$\frac{f(1) - f(-1)}{1 - (-1)} = \frac{1 - 1}{2} = 0.$$

Is there some c in the interval $(-1, 1)$ for which $f'(c) = 0$? Why does this not contradict MVT?

Exercise 4. On a previous homework you encountered the *greatest integer* function $f(x) = \lfloor x \rfloor$, which denotes the largest integer not exceeding x.

Its derivative $f'(x)$ is equal to 0 (where defined). But f is certainly not a constant function. Why does this not contradict the Corollary to MVT?

Hopefully the preceding problems have given you some feeling for when the various theorems are applicable. Now try your hand at using the IVT and MVT to solve the following problem.

Problem. Show that there is exactly one positive value of x satisfying the equation

$$x^4 + 4x^2 = 4x^3 + 8.$$

Suggestion: use IVT to argue that there is *at least one* value of x satisfying the equation. Then find a way to argue using MVT that there is *at most one* value of x satisfying the equation.